This is an essay introducing the Philosophy of Mathematics. It is not about Math itself or doing Math, which I confess I am not well versed. Conversely, I find few students of math who are well versed in the Philosophy behind their subject. They just do the math! But, if Christians are to do Math, thinking as a Christian, understanding what they do from a Biblical Perspective, knowledge of the philosophy (or assumptions or faith) that under-girds and shapes "doing math" is essential.

We are challenged to see Christ and His Word as Sovereign over every academic subject. Many insist that academics may (and should) be done autonomously and impartially. Fact is, they are never done apart from faith statements, assumptions and biases of some kind. Nothing is neutral and completely objective. We believe the beginning of all knowledge is the fear of the Lord. I admit that to be my presupposition as I approach every Subject. True truth can be known only when a subject is studied in relation to God and what He has revealed about reality, this includes Mathematics. This essay will begin to study Math in light of the Bible and Christian Theology. It should be read in the light of my paper on Epistemology (also on this site). The two subjects are inseparable.

The following are [slightly revised] lecture notes from a Class I offered for college students in the Winter of 1979. Since that time, of course, Constructivism and Post Modernism have come on the scene and there have been new approaches to Theories of Knowledge, Logic and Math. I know these lecture notes need revision and updating, but they are still relevant and worthwhile for understanding what Math is about. They are also relevant to the current theological hot topic of Open Theism.   **BEGINNING A BIBLICAL PHILOSOPHY OF MATHEMATICS**

Unit I: Arithmetic

1. Properties of Numbers

2. Properties of Arithmetical Operations, eg- distribution, commutation, association, aggregation (unity and plurality), natural /rational vs. irrational

3. Reasoning by Recurrence, Complete Induction, principle of Permanence

Unit II: Infinity

1. Special Topics: Time, Continuity, Succession

Unit III: Foundations of Modern Mathematics

1. Formalism

2. Logicism

3. Intuitionism

4. Constructivism

Unit IV: Causality and Philosophy of Science [This is a separate Essay elsewhere in this Section of the website]

Bibliography: Beside the Scripture, the main sources for this paper, written in 1979, were

Non-Christian-

Math in Western Culture, Morris Kline

Number, Tobias Dantzig

Math for the Modern Mind, Walter Fuchs

New World of Math, George Boehm

Mathematics, David Bergammi

Mathematics for Parents, Carl Doerfer

The Common Sense of the exact Sciences, Wm Clifford

Christian-

Christian Perspectives on the Foundations of Math (a series of Papers presented over several years at a Conference for that purpose sponsored and published by Wheaton College)

Foundations of Christian Scholarship (chap 8) by Vern Poythress

UNIT 1- ARITHMETIC: NUMBERS (Basics)

A. Definitions-

1. Arithmetic is the measurement of quantitative and spatial
   (geometric) aspects of natural Phenomena

   Main characteristics: abstraction, symbols, logic (axioms,
   postulations, proofs and deductions)

2. Numbers were originally things used to count other things
   by correspondence

    a. Natural numbers were originally things, then
       pictures, words, symbols and abstractions

    b. Counting involves succession, sequence, infinity and
       concepts of continuity and motion

    c. Correspondence involves concepts of Truth, Reality and
       Knowledge

  3. Number Fields: Natural, Rational, Real and Complex; also
    Irrational, transcendental and Infinitesimal

These topics raise philosophical issues and presuppositions relevant today in many areas.

B. Four Fundamental Operations with Natural (Whole) Numbers

    1. Addition

    2. Multiplication

    3. Subtraction

    4. Division

These all require certain properties-

C. Three Basic Properties of Operations

    1. Distribution- multiplication is really repeated addition
       (for positive integers)

    2. Commutation- two whole numbers may be interchanged

    3. Association

D. More Arithmetical Philosophical Problems

    1. Universals and Particulars (One and the Many or Unity
       and Plurality)

    2. Continuity and Permanence (of natural sequences)

    3. Infinity: a never ending series

Biblical and Theological Material Relevant to the Philosophical Issues raised so far (above)-

UNITY II- INFINITY

The basic assumption about infinity in math is that there can be an unending succession. Consider a series of dots, making a line, that has no end. This is a way of defining Time

   A. Time in the Bible: Key Words

      1. (Heb) Olam - translated in English as "ever" (272x)
          or "everlasting" (63x) in KJV

      2. (Grk) Aion - translated as "eternal" (42x) or
          "everlasting" (25x) in KJV

          Aion means-

          a. segment of time, often used in the plural; an
             unending (infinite) series of ages

             Col 1:26, Eph 3:11, Heb 9:26, I Cor 10:11

          b. a long period of time: this age or the age to come

          c. Eternity

          d. the world as the natural order [this usage
              connects time and space]

      3. (Grk) Kairos- translated as "Time" - a definite fixed
          period of time (87x) KJV

      4. (Grk) Chronos- translated as time- a period of time of
          indeterminate length (53x) KJV

   B. Doctrine of Time - there will never be an end to all time,
       i.e.- a timeless Eternity

      1. Created- Gen1:1, Jn 1:1-2, Mt 19:4-8, Heb 1:10, Heb
          1:2, 11:3; 1 Tim 1:17 cf Ps 66:7, Ex 15:18

      2. Temporary- (this Age) Mt 13:39-40, 24:13, 28:20

3. Recreated- (the Age to come, the new Age Lk 18:30

4. The Sphere in which God works redemptively. The
    Incarnation and Atonement, et al happened within
    time. Redemption is temporal.

    Jn 2:4;7:6,8,30; 8:20; 12:23,27; 13:1; 17:1,4; 19:30

5. The Center of Time is the Christ Event. It is now
    past, but it governs all History

6. God's Providence works within time (in History)

7. Time is directed by God to a intelligent and moral
    Goal; Time fulfills God's Purpose- Eph1:9-11

C. The Biblical Meaning of Eternity

    The English word appears only one time - Isa 57:15
    (KJV). Heb "ad" is translated as "ever" 41x

    See the words Olam and Aion above. The latter is
    translated as Eternity. Eternity means infinite
    time, ie- time without end

D. The Eternality of God [not the timelessness of God]-
    Deut 33:27a, Isa 57:15a, Rom 16:26 cf 1:20, Ps 29:10,
    45:6 etc

    1. Eternality is required by God's nature: a spirit,
        perfect, omniscient and omnipresent Although God
        works within time, He transcends all temporal limits
        (duration, succession, division). His nature and
        attributes have no limits at all.

    2. Olam and Aion are used in the plural in reference to
        God (ie- Age to Age, without end)

    Gen 21:33, Deut 33:27b, Ps 90:2, 4 cf 2 Pet 3:8; Ps
    93:2, 102:24,27; Isa 26:4, 40:28, 44:6; Heb 1:8-12,
    13:8; Rev 1:4,8,17; 2:8; 4:8; 21:6;22

The Mathematical Concept of Infinity, of a never ending series, is rightly rooted in the Nature of
God. It is Biblical and True. Aion follows Aion without end. God is all God at every point on

the line, forever. This Concept does not come from intuition or induction- it comes directly from God.

Ecclesiastes 3:11 [God] has put eternity [Olam= a sense of the everlasting cf Chronos, a sense of indefinite or perpetual Time] into man's heart. (This also means Humans "know" their life is not over at the grave.)
Continuum: "a well-ordered and perfect aggregate" (the set of all points on a line segment)

"well-ordered" = all numbers permanently arranged in greater and lesser sequence

"perfect" = contains its own limiting values from negative infinity to positive infinity

"aggregate" = sum or collection (a group or set)

Numbers that can be represented on a line- ["real" numbers are believed (intuited) to be in one to one correspondence with the points of the number line, in contrast to imaginary numbers)

Whole (natural or counting) numbers: (positive integers 1 2 3 or negative: -1 -2 -3 or zero)

Rational Numbers: fractions

Irrational Numbers: not fractions, all real Numbers that are not Rational

Some Philosophers (such as Danzig) say that the concept of Continuum, as well as Infinity, rest on Intuition. Furthermore, the mind connects Continuum with Causality, ie- it rejects the idea of chance and spontaneity and insists on necessity (if 3 than 4). The Continuum requires belief in necessary connection, not just constant conjunction. There must be a chain of cause and effect.

Biblical basis for Continuum, as well as Infinity, is the Doctrine of Time and Causality (above)

Universals and Particulars (One and the Many or Unity and Plurality)

   A. There are 3 traditional and non-Biblical solutions to this
       problem.

       a. All is One - differences are illusionary

       b. All is many - with no necessary connections

       c. Particulars are related by Universals (or Ideals or
           Absolutes) Particulars derive their individual
           meaning by being connected with, a part of, their
           Universal

The man is Human. Humanness is Universal. The man is a particular

Furniture is a Universal. Chairs and tables and stools are particulars.

The entire 500 piece jigsaw puzzle is a Universal. Each piece is a particular

A sentence is a Universal. Subjects and predicates are particulars

B. Questions re Universals

   a. Do Universals exist?

   b. What accounts for the concept of (the belief in) Universals?

   c. Can we truly know Universals

   d. Is this topic simply a matter of semantics?

   e. in Math, are numbers Universals or Particulars?

C. Biblical Solution to the Problem of the One and Many

Particulars are all related in a cause and effect chain and by God's Design or Plan for them. The relationship of the One and Many is explained by the Biblical Doctrine of Causality and Providence. The source of Unity (and meaning) is not in Universals, but in Christ: All things, that are other than Deity, hold together in Him -Col 1:17 These "things" are in no way divine, or part of, Deity.

The tri-unity of God is a model of particulars in unity; but, these particulars (Father, Son and Spirit) are equals in nature and not in any kind of cause and effect chain. In the economy of God, even the "roles" of each "Person" are of equal importance and work together.

The Problem of Correspondence and Truth

We earlier defined "numbers" as "things" that correspond to other things. What is "correspondence"? This is very important as correspondence is one of two definitions of "truth". The other is "coherence".

Correspondence is assigning every object of one collection and object of another, eg- rational numbers to points on a line [the basis of Analytic Geometry]. One thing is the arbitrary, fixed, representation of the other. (Representation does not mean one looks like the other, but it symbolizes it.) The problem is- does what a symbol is said to represent, actually exist independent of its symbol? Is mathematical truth really "true"?

Earlier, we defined "Arithmetic" as "the measurement of quantitative and spatial (geometric) aspects of natural phenomena." Arithmetic is an attempt to explain reality. Problems developed when arithmetic seemed to fail to explain certain aspects of reality, eg- irrational numbers, transcendentals and infinite numbers. How real are imaginary and complex numbers (numbers who's square is negative or which are a combination of real and imaginary numbers) Do such thing really exist apart from their assigned symbols? What is the relationship of Analytic, algebraic, descriptive, differential, projective and non-Euclidean geometries to each other. Which one truly represents "reality"?

Modern Math does not attempt or claim to represent or explain reality. It is its own reality! Traditionally, the process of constructing math theory had four operations: [Nature/Reality => Intuition => Organization => Deduction => Application [to Nature and the real world].

In 1965, C. Allendoerfer, Professor of Mathematics at the University of Washington said, (mathematical models) "are abstract systems of undefined words, axioms, etc (that) need have nothing whatsoever to do with nature. They are no longer models of anything, but are merely structures built by mathematicians because they are thought worth investigating. Since these abstract mathematical systems did not spring from nature, there is no obligation for their advocates to apply related theorems back to nature and consequently we have a mathematics which exists purely of and for itself. A large part of contemporary math is of this kind."

Modern mathematicians are not interested in Truth or non-Truth at all. They are interested only in validity, internal consistency and non-contradiction.

All of this leads the Christian to ask "What is reality? Can math explain it? Must math correspond to it? Must math be concerned with Truth? Can a Christian engage in a field which claims complete autonomy from reality and truth? How does a Christian, who believes in the reality of Creation and its Creator, who claims sovereignty over all that is, and who, as a Christian, believes the purpose of all that is, including the designs and products of out human minds, is to glorify God, respond to this-

"Math has a reality of its own. It is a reality that is defined in the human maind and need not be confirmed by the interpretation of a physical experiment. The physicist is fettered to whatever

his instrument tell him is real; the mathematician, however, is free to explore a much richer world, mostly of his own making." - G. Boehm

"How then shall mathematical concepts be judged? They shall not be judged. Mathematics is the supreme judge. From its decisions there is no appeal" -Dantzig

Most researchers in math today apparently would agree with C.G. Jacoby who, when asked why he did math, replied: "to glorify the human intellect." -Boehm

The arrogant hubris of these quotes, from more than some 30 years ago, simply takes my breath away and offends me greatly as a Christian. The attitude has spread to many non-math fields (including Theology) and is even more prevalent today. Consider those statements in light of-

1 Corinthians 10:31...whatever you do, do all to the glory of God.

2 Corinthians 10:5 … take every thought captive to obey Christ

Colossians 1:15 [Jesus Christ] is the image of the invisible God, 16 by him all things were created, in heaven and on earth, visible and invisible, --all things were created through him and for him. 17 And he is before all things, and in him all things hold together. 18 … that in everything he might be preeminent.

Nothing, including Mathematics can be autonomous. Whatever Reality and Truth are, math must take its place within them and under the authority of Christ. Mathematics must be done for Him.

UNIT III: Foundations of Modern Mathematics

A. Formalism (David Hilbert, et al)

1. Axioms are meaningless symbols, devoid of all meaning

2. Math consists of the manipulation of arbitrary symbols according to arbitrary rules

3. Math is mental Chess game, a purely mental creation

4. Math is built on formal axioms without regard to truth content (results have no signifigance- only getting them does)

5. Math is self-contained, totally autonomous. It says nothing of the world

6. Hilbert sought to establish mathematics without

contradictions or paradox and with absolute certainty- all based on the techniques of logical proof

Major Weakness:

1. Godel's use of the Liar's Paradox scuttled Hilbert's efforts. He proved that no formal system can be complete and consistent-

   "This sentence is false" or The sentence on side 2 is true. The sentence on side I is false."

 2. Formalism does not deal with ontology- it can not explain how math can be applied, eg- in Physics

B. Logicism (B Russell and A.N. Whitehead)

  1. Axioms are arbitrary

  2. What specific axioms say is irrelevant (any consistent set will do). Math is concerned with what follows the axioms

  3. Logicism is similar to Formalism, but with more emphasis on Logic. Math is Logic

  4. Math is a matter of symbolizing and recording certain patterns of reasoning (Arguments)

  5. Arguments are created at will. There is no relationship to experience

  6. Math is the art of transforming a reasoned argument into mathematical terminology

Major Weakness: though stressing Logic, this school uses many unproven assumptions in its arguments (eg- the properties of Arithmetic)   It gives no justification for what it does.

C. Intuitionism (H. Bower)

  1. Axioms are hypotheses, same as in the physical sciences

  2. Math is a system of hypotheses about numbers based on various evidence

3. Mathematical truth is objective: determined by usefulness in explaining data

4. Mathematical axioms are tested and sometimes abandoned on the basis of evidence

5. Intuitionism rejects the concept of Infinity, the Continuum and all set theory

6. Intuitionism rejects value logic (ie- the Law of Excluded Middle)- same statements may be True or False or Undecidable

Major Weakness:

1. The rejections of concepts based in Scripture and the Nature and character of God

2. It can not account for success in applying math to the "real" world

D. Constructivism (1979)- General Summary Re. Mathematics

1. Self-contained, autonomous, mental chess game

2. Built on formal axioms which are arbitrary symbols

3. No concern with truth content- says nothing about the "real world"

4. Denies Universals. Only particulars exist and can be known

5. Denies a priori knowledge

6. Denies that anything exists beyond the sensory world

7. Thoroughly Empirical

8. All knowledge is posteriori (only the observable is knowable)- all knowledge of matters of fact must be founded on observation and memory and all reasoning is by induction from observation and memory

Conclusion:

It should be readily seen that much of the common Philosophy of Mathematics is in opposition to Scripture and Reformed Christian Epistemology. Here is a challenge to the Christian Mathematician who wants to pursue this field pro rege. Develop a Philosophy of Mathematics that is Biblical.